Hydroelasticity of a Large Floating Wind Turbine Platform

TOBIAS FINN
Hydroelasticity of a large floating wind turbine platform

Author: Tobias Finn
tfinn@kth.se

Supervisor: Marcus Thor
Examiner: Anders Rosén

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Abstract

This thesis defines a limit for when hydroelasticity is necessary to include in an analysis of a large floating semi-submersible wind turbine platform in waves. The thesis also includes a description of how to include hydroelasticity in the design of such a structure. A simple analysis studying two two-dimensional beams’ hydroelastic behaviour in waves is also conducted, observing resonance, large deformations and stresses in the vicinity of the first elastic natural frequency.

Hydroelasticity concerns the combined fluid-structure interaction for floating flexible structures in waves. In a hydroelastic analysis the fluid forces and structural deformations are coupled to account for dynamic and kinematic effects. In this thesis the analysed structure is assumed to be beam-like and Euler beam theory is used. The hydrodynamic forces are determined using a linearised Morison’s equation. The hydroelastic response is performed in the frequency domain using a modal analysis and it is modelled in a self-developed model using Matlab.

Most of the concepts and prototypes of floating wind turbines of today have one turbine installed on a floater and the structure is assumed to be rigid. When modelling a structure as flexible, elastic responses is observed around the elastic natural frequencies.

The analysis has been performed on two beams with different lengths and stiffness’ to observe a hydroelastic behavior: 1) when the first wet elastic natural frequency is about four times the peak frequency of the sea spectra and 2) when the first wet elastic natural frequency is almost within the sea spectra.

It has been found that if the first wet elastic natural frequency of the structure is higher than about 2-5 times than the wave frequency in regular waves or about five times the peak frequency, a quasi-static assumption is reliable. If the first wet elastic natural frequency is less than that, hydroelasticity needs to be considered. The actual limit for a quasi-static/hydroelastic assumption needs to be further investigated.
Sammanfattning

Den här uppsatsen definierar en gräns för när hydroelasticitet är nödvändig att inkludera i en analys av en stor flytande semi-submersible vindkraftverksplatform i vågor. Uppsatsen beskriver också hur hydroelasticitet kan inkluderas i konstruktionen av en sådan struktur. En förenklad analys har gjorts där två tvådimensionella balkars hydroelastiska beteende i vågor har studerats. I den observerades resonans, stora deformations och stora spänningar omkring den första elastiska egenfrekvensen.

Hydroelasticitet är den kombinerade fluid-struktur-interaktionen för en flytande flexibel struktur i vågor. I en hydroelastisk analys är fluidkrafterna och strukturdeformationerna kopplade för att ta hänsyn till dynamiska och kinematiska effekter. I denna uppsats antas den analyserade strukturen vara balklik och Euler balkteori har använts. De hydrodynamiska krafterna bestäms m.h.a. en linjäriserad Morisons ekvation. Det hydroelastiska gensvaret har beräknats i frekvensplanet m.h.a. en modal analys och det har modellerats i en egenutvecklad modell i Matlab.

De flesta koncept och prototyper för flytande vindkraftverk har idag monterat en turbin på flytkroppen och strukturen har antas vara stel. När en struktur modelleras som flexibel observeras ett elastiskt gensvar omkring de elastiska egenfrekvenserna.

Analysen har gjorts på två balkar med olika längd och styvhet för att observera ett hydroelastiskt beteende 1) när den första våta elastiska naturliga egenfrekvensen är ungefär fyra gånger peak-frekvensen av sjöspektrat och 2) när den första våta elastiska naturliga frekvensen nästan ligger inuti sjöspektrat.

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<td>Stochastic linearisation</td>
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## Nomenclature

### Abbreviations

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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>ANSYS Aqwa</td>
<td>Wave analysis extension program developed by ANSYS</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CIP</td>
<td>Constrained Interpolation Profile</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>d.o.f.</td>
<td>degree of freedom</td>
</tr>
<tr>
<td>DNV</td>
<td>Det Norske Veritas - a classification society</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>GVA</td>
<td>a consultancy company providing engineering services within the offshore industry</td>
</tr>
<tr>
<td>ISSC</td>
<td>International Ship and Offshore Structures Congress</td>
</tr>
<tr>
<td>JONSWAP</td>
<td>Joint North Sea Wave Project</td>
</tr>
<tr>
<td>KTH</td>
<td>Kungliga Tekniska Högskolan</td>
</tr>
<tr>
<td>Matlab</td>
<td>a programming language and program</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds Averaged Navier-Stokes</td>
</tr>
<tr>
<td>RAO</td>
<td>Response Amplitude Operator</td>
</tr>
<tr>
<td>SPH</td>
<td>Smooth Particle Hydrodynamics</td>
</tr>
<tr>
<td>SWATH</td>
<td>Small Waterplane Twin Hulls</td>
</tr>
<tr>
<td>swl</td>
<td>stillwater level</td>
</tr>
<tr>
<td>TLP</td>
<td>Tension Leg Platform</td>
</tr>
<tr>
<td>WADAM</td>
<td>a wave analysis program developed by DNV which is based on the theory of earlier versions of WAMIT</td>
</tr>
<tr>
<td>WAMIT</td>
<td>a wave analysis program based on potential flow theory developed by Prof. J.N. Newman and Dr. Chang-Ho Lee at MIT, Massachusetts, USA</td>
</tr>
<tr>
<td>VLFS</td>
<td>Very Large Floating Structure</td>
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## Symbols

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<th>Description</th>
<th>Unit</th>
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<tr>
<td>(a)</td>
<td>mass</td>
<td>tonnes</td>
</tr>
<tr>
<td>(A_n)</td>
<td>structural and added mass for mode n</td>
<td>tonnes</td>
</tr>
<tr>
<td>(A^m)</td>
<td>modal mass</td>
<td>tonnes (\cdot) m</td>
</tr>
<tr>
<td>(A_{ref})</td>
<td>reference area</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(A_w)</td>
<td>area about the stillwater level</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(A^w)</td>
<td>hydrodynamic added mass</td>
<td>tonnes</td>
</tr>
<tr>
<td>(A_{xs})</td>
<td>cross sectional area of the beam</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(b)</td>
<td>damping</td>
<td>tonnes/s</td>
</tr>
<tr>
<td>(B_n)</td>
<td>hydromechanical/structural damping for mode n</td>
<td>tonnes/s</td>
</tr>
<tr>
<td>(B^m)</td>
<td>modal damping</td>
<td>tonnes (\cdot) m/s</td>
</tr>
<tr>
<td>(B^w)</td>
<td>hydrodynamic damping</td>
<td>tonnes/s</td>
</tr>
<tr>
<td>(c)</td>
<td>stiffness</td>
<td>tonnes/s(^2)</td>
</tr>
<tr>
<td>(C_a)</td>
<td>added mass coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>(C_d)</td>
<td>drag coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>(C_n)</td>
<td>hydromechanical/structural stiffness for mode n</td>
<td>tonnes/s(^2)</td>
</tr>
<tr>
<td>(C_m)</td>
<td>inertia coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>(C^m)</td>
<td>modal stiffness</td>
<td>tonnes (\cdot) m/s(^2)</td>
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<tr>
<td>(C_w)</td>
<td>hydrodynamic stiffness</td>
<td>tonnes/s(^2)</td>
</tr>
<tr>
<td>(d, D)</td>
<td>diameter of the node</td>
<td>(m)</td>
</tr>
<tr>
<td>(E)</td>
<td>Young’s modulus</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(F, P)</td>
<td>force in general</td>
<td>(N)</td>
</tr>
<tr>
<td>(dF^\text{wave}_x)</td>
<td>horizontal force per unit length</td>
<td>(N/m)</td>
</tr>
<tr>
<td>(F^{\text{elast}})</td>
<td>forces due to the elastic motion</td>
<td>(N)</td>
</tr>
<tr>
<td>(F_D, F_{FK}, F_\zeta)</td>
<td>drag-, Froude-Krylov-, stiffness force term</td>
<td>(N)</td>
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<tr>
<td>(F_{\text{Morison}})</td>
<td>Morison’s equation</td>
<td>(N)</td>
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<td>(F^m)</td>
<td>modal wave forces</td>
<td>(Nm)</td>
</tr>
<tr>
<td>(F_{\text{moor}})</td>
<td>mooring force</td>
<td>(N)</td>
</tr>
<tr>
<td>(F^{\text{rigid}})</td>
<td>forces due to the rigid body motion</td>
<td>(N)</td>
</tr>
<tr>
<td>(F^\text{wave})</td>
<td>wave forces</td>
<td>(N)</td>
</tr>
<tr>
<td>(g)</td>
<td>gravitational constant</td>
<td>(m/s^2)</td>
</tr>
<tr>
<td>(G_n)</td>
<td>transfer function</td>
<td>(s^2/\text{tonnes})</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>$h$</td>
<td>water depth</td>
<td>$m$</td>
</tr>
<tr>
<td>$H, H_s$</td>
<td>wave height, significant wave height</td>
<td>$m$</td>
</tr>
<tr>
<td>$i$</td>
<td>the imaginary number</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>second moment of area</td>
<td>$m^4$</td>
</tr>
<tr>
<td>$j$</td>
<td>index for local d.o.f.</td>
<td></td>
</tr>
<tr>
<td>$J$</td>
<td>number of element divisions</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>wave number</td>
<td>$1/m$</td>
</tr>
<tr>
<td>$k_\theta$</td>
<td>rotational stiffness</td>
<td>$Nm/m$</td>
</tr>
<tr>
<td>$k_x, k_y, k_z$</td>
<td>translational stiffness in x-, y- and z-direction</td>
<td>$N/m$</td>
</tr>
<tr>
<td>$K(\sigma_u)$</td>
<td>linearisation coefficient</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$K_C$</td>
<td>Keulegan-Carpenter number</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>length of beam</td>
<td>$m$</td>
</tr>
<tr>
<td>$m$</td>
<td>total mass</td>
<td>$tonnes$</td>
</tr>
<tr>
<td>$M$</td>
<td>bending moment</td>
<td>$Nm$</td>
</tr>
<tr>
<td>$M^\text{wave}_\theta$</td>
<td>wave moment</td>
<td>$Nm$</td>
</tr>
<tr>
<td>$n$</td>
<td>index for global d.o.f.</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>number of modes</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>shear force</td>
<td>$N$</td>
</tr>
<tr>
<td>$S, S_{ISSC}$</td>
<td>Bretschneider and ISSC sea spectra</td>
<td>$m^2s$</td>
</tr>
<tr>
<td>$t$</td>
<td>time variable</td>
<td>$s$</td>
</tr>
<tr>
<td>$T$</td>
<td>draught of a node</td>
<td>$m$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>eigenperiod</td>
<td>$s$</td>
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<tr>
<td>$T_L, T_p, T_z, T_w$</td>
<td>loading-, peak-, zero crossing- and wave period</td>
<td>$s$</td>
</tr>
<tr>
<td>$u$</td>
<td>water particle velocity</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$\dot{u}$</td>
<td>water particle acceleration</td>
<td>$m/s^2$</td>
</tr>
<tr>
<td>$V$</td>
<td>volume of submerged body</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$x$</td>
<td>horizontal direction</td>
<td>$m$</td>
</tr>
<tr>
<td>$z$</td>
<td>vertical direction</td>
<td>$m$</td>
</tr>
<tr>
<td>$z_{beam}$</td>
<td>height of the beam</td>
<td>$m$</td>
</tr>
<tr>
<td>$z_{NA}$</td>
<td>distance from the stillwater level to the neutral axis</td>
<td>$m$</td>
</tr>
<tr>
<td>$Z$</td>
<td>displacement vector for all local d.o.f</td>
<td>$m$</td>
</tr>
<tr>
<td>$Z_m$</td>
<td>modal displacement vector for all local d.o.f</td>
<td>$m \cdot m$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>α, β</td>
<td>coefficients in the Bretschneider spectra</td>
<td>−</td>
</tr>
<tr>
<td>βn</td>
<td>coefficient for calculating eigenfrequencies</td>
<td>−</td>
</tr>
<tr>
<td>δ_{\text{elastic}}</td>
<td>deflection for the elastic beam</td>
<td>m</td>
</tr>
<tr>
<td>δ_{\text{hinge}}</td>
<td>deflection for the hinged beam</td>
<td>m</td>
</tr>
<tr>
<td>ζ</td>
<td>effective wave amplitude</td>
<td>m</td>
</tr>
<tr>
<td>ζ_a</td>
<td>wave amplitude</td>
<td>m</td>
</tr>
<tr>
<td>θ</td>
<td>pitch angle</td>
<td>rad</td>
</tr>
<tr>
<td>λ</td>
<td>wave length</td>
<td>m</td>
</tr>
<tr>
<td>ξ</td>
<td>damping factor</td>
<td>−</td>
</tr>
<tr>
<td>ρ</td>
<td>density of salt water</td>
<td>tonnes/m³</td>
</tr>
<tr>
<td>σ</td>
<td>stress</td>
<td>Pa</td>
</tr>
<tr>
<td>σ_u</td>
<td>standard deviation of the water particle velocity</td>
<td>m/s</td>
</tr>
<tr>
<td></td>
<td>of a given irregular sea response spectra</td>
<td></td>
</tr>
<tr>
<td>τ</td>
<td>shear stress</td>
<td>Pa</td>
</tr>
<tr>
<td>φ</td>
<td>matrix containing of mode shapes in the columns</td>
<td>m</td>
</tr>
<tr>
<td>Φ</td>
<td>velocity potential</td>
<td>m²/s</td>
</tr>
<tr>
<td>ω, ω_e, ω_L, ω_n</td>
<td>wave-, eigen-, loading- and natural frequency</td>
<td>rad/s</td>
</tr>
<tr>
<td>ω_{\text{limit}}</td>
<td>limit frequency of where the loading condition may be assumed quasi-static/hydroelastic</td>
<td>rad/s</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Thesis background

This master’s thesis within Naval Architecture at the Royal Institute of Technology (KTH\textsuperscript{1}) has been performed at Hexicon AB. Hexicon is a company founded in 2009 and their business idea is to develop concepts of floating wind turbine platforms containing three or more turbines. To be able to install three or more turbines these structures need to be very large.

Within the shipping and offshore industry the standard way of calculating the hydrodynamic loads are using rigid-body assumptions, but with large and slender structures, such as Hexicon’s, there is a point where rigid body assumptions are no longer applicable and the flexibility (hydroelasticity) of the structure needs to be included in the analysis.

After a report issued by GVA\textsuperscript{2} analysing one of Hexicon’s platforms, the question of hydroelasticity was raised. GVA analysed the structure using a semi-flexible analysis, which was an attempt to recreate a flexible structure with the use of rigid interconnected bodies.

This thesis will try to define if hydroelasticity is of concern for one of Hexicon’s platforms and to increase the knowledge within hydroelasticity for Hexicon. The thesis outline is two separate literature studies, a simplified hydroelastic analysis, concluding remarks and an appendix containing relevant theory. The first literature study concerns the degree of hydroelasticity, i.e. when hydroelasticity becomes a concern of a floating structure. The second literature study is about floating wind turbines of today; by which methods they have been modelled accordingly, what classification societies has to

\textsuperscript{1}Kungliga Tekniska Högskolan

\textsuperscript{2}GVA is a consultancy company providing engineering services within the offshore industry.
say about floating wind turbines and a review of the report issued by GVA. The simplified hydroelastic analysis concerns a structure similar to one of Hexicon’s structures, determining the degree of hydroelasticity, the preliminary deformations and stresses of it. The concluding remarks summarize the conclusions from each previous chapters, it summarizes what has been done and discusses the different assumptions and the validity of those. The appendix presents a more detailed description of the theory used in the case study.

1.2 Objectives

The objectives of this thesis is to increase the knowledge within hydroelasticity with the main focus on floating wind turbines by:

- review how scientists nowadays perform hydroelastic calculations,
- review how scientists determine the wave loads and structural motions on floating wind turbines of today
- review the market for possible software’s where hydroelasticity may be incorporated,
- defining when a quasi-static analysis is insufficient and a hydroelastic analysis needs to be performed and
- describe the methodology provided by DNV\textsuperscript{3} when designing a floating wind turbine, including hydroelasticity.

In order to get an idea of the hydroelastic behaviour of one of Hexicon’s platforms and to highlight some problematic areas where more work needs to be done, the thesis will also include:

- a simple analysis of a structure similar to Hexicon’s platforms where hydroelasticity is included and
- a critical review of the report by GVA of one of Hexicons platforms.

\textsuperscript{3}Det Norske Veritas - a classification society
Chapter 2

Literature study: Degree of hydroelasticity

2.1 Introduction

Very large floating structures, commonly referred to as VLFS\textsuperscript{4}, are being built to e.g. accommodate floating airports, the world’s largest container ships reaching lengths in excess of 400 meters and concepts of floating wind platforms having dimensions exceeding 500 meters. As these structure gets longer and more slender and thus more flexible, hydroelasticity (which is the study of flexible bodies in waves) may be of concern.

Research of hydroelasticity has been ongoing since the 1970s with Bishop and Price (1979) as pioneers within this field. A lot of the research within hydroelasticity has been performed on VLFS’s with the objective to be used as oil storage, to accommodate residences or airports e.g. the floating airport in the bay of Tokyo, Mega Float (SRCJ, 2013).

A hydroelastic analysis is more comprehensive than a quasi-static analysis, which is commonly used in the industry, and it is thus very beneficial of performing a quasi-static analysis instead of a hydroelastic analysis if possible. The subsequent question when a hydroelastic analysis is required, is raised. This chapter will investigate when a hydroelastic analysis must be performed over a quasi-static in a literature study and a parameter study based on the model described in chapter 4.

\textsuperscript{4}Very Large Floating Platform
2.2 About hydroelasticity

Hydroelasticity concerns the combined fluid-structure interaction analysis for flexible structures in waves. In a hydroelastic analysis the fluid forces and structural deformations must be performed at the same time, called a coupled analysis. A good description of hydroelasticity is a quote by Bishop et al. (1986):

"hydroelasticity is the study of the behaviour of a flexible body moving through a fluid."

In the case of a stationary body, "moving through a fluid" may be interpreted as a moving fluid acting on the stationary body. Bishop and Price are pioneers within this field of research with their book "Hydroelasticity of Ships" by Bishop and Price (1979). The book concerns linear hydroelastic theory which originates in structural dynamics and is applied to bodies moving through a fluid. The phenomenon of hydroelasticity is analogous to aeroelasticity in the aerospace industry.

2.3 Modelling hydroelasticity

There are several modelling techniques to perform hydroelastic analyses with, all with its own level of sophistication. Some of which are described below. There are also different levels of detail that may be applied in the analysis. Global hydroelasticity refers to the entire structure being subject of an analysis, where the level of detail may be low yet still obtain reliable results. In contrast to that, local hydroelasticity is where the focus is on an individual part or a substructure, and where the level of detail needs to be high in order to obtain a reliable result.

2.3.1 Global hydroelasticity

A typical global hydroelastic analysis is on an entire ship or a VLFS. Linear hydroelastic theory, described in (Bishop and Price, 1979) is the most simple type of global analysis and is also used herein. The structure is assumed to be beam-like and Euler beam theory is used. Alternatively Timoshenko beam theory may be used, which takes into account shear deformations and rotational inertia effects (Zenkert, 2005). This theory is more comprehensive and is appropriate for short beams or sandwich beams with a low shear
stiffness. The hydrodynamic coefficients are determined using strip theory i.e. two-dimensional potential flow, which is common in the shipping industry. In the offshore industry Morison’s equation is widely used for this. Morison’s equation describes the oscillating wave force on circular cylinders which are slender in relation to the wave length.

A more advanced way is to use a wave analysis program based on potential theory (such as the commercial softwares WAMIT\textsuperscript{5} or ANSYS Aqwa\textsuperscript{6}). These programs uses a panel method to determine the hydrodynamic coefficients and the wave forces. The deformations of the structure are determined in a FE\textsuperscript{7}-model. Here, a three-dimensional hydroelastic theory may be used, which is an extension of the two-dimensional theory that handles the added third dimension.

It is possible to perform these calculations either in the time or frequency domain. Both of these types of analyses have their respective pros and cons. In the time domain, a direct approach is carried out, which calculates the forces and deformation for each time-step. This is intuitive but CPU demanding. In this approach one may take into account non-linear effects such as the non-linear drag force or non-linear waves, which may be of importance. In the time domain one may apply a more advanced CFD\textsuperscript{8}-solver to account for viscous flows. In the frequency domain a solution is obtained for each frequency of interest and a modal analysis is carried out. This reduces the CPU-power significantly, but non-linear effects must be linearised in order to be taken into account.

2.3.2 Local hydroelasticity

Local hydroelasticity refers to an individual part or substructure being subject of analysis. This has not been the main focus in this thesis and is only described briefly. Some cutting-edge techniques for hydrodynamic calculations to solve viscous flows in marine applications are ALE\textsuperscript{9}, RANS\textsuperscript{10}, SPH\textsuperscript{11} and CIP\textsuperscript{12} methods, which are able to capture

\textsuperscript{5}WAMIT is a wave analysis program based on potential flow theory developed by Prof. J.N. Newman and Dr. Chang-Ho Lee at MIT, Massachusetts, USA

\textsuperscript{6}ANSYS Aqwa is a wave analysis program based on potential flow theory in the ANSYS product suite

\textsuperscript{7}Finite Element

\textsuperscript{8}Computational Fluid Dynamics

\textsuperscript{9}Arbitrary Lagrangian-Eulerian

\textsuperscript{10}Reynolds Avaraged Navier-Stokes

\textsuperscript{11}Smooth Particle Hydrodynamics

\textsuperscript{12}Constrained Interpolation Profile
non-linear effects and/or violent flows i.e. slamming. For global applications, i.e. entire structures, these methods are not manageable due to their high demand in CPU-power. These methods also provide information on the fluid flow around the structure which is irrelevant when determining the global forces on the hull. These methods are more applicable at local analyses such as slamming and may be coupled to a FE-model for a hydroelastic analysis. Hydroelasticity on panels of a high speed craft subjected to slamming forces has been studied previously by Stenius et al. (2011) and hydroelasticity of foils have been studied by Feymark (2013). This is on a local level and advanced CFD-solvers coupled with an FE-solver is a necessity to capture these non-linear effects. Alternatively, the explicit fluid-structure ALE method may be used, where both the fluid and structure is modelled in the same domain.

Vortex induced vibrations may be a source of local hydroelasticity and must be analysed appropriately.

2.3.3 Market for hydroelastic calculations

In the present situation, there is no individual commercial tool on the market which is able to perform global hydroelastic calculations independently i.e. without coupling to another software. With the use of the ALE method is this theoretically possible, but the CPU-power needed would be enormous. However, ANSYS have indicated that their product is able to do this with the use of Morison type elements (ANSYS, 2013), neglecting diffraction and radiation. Researchers within this field have extensively used the wave analysis program Wamit coupled with a commercial FE-solver (Taghipour, 2008),(Kim et al., 2012). The particular use of Wamit in hydroelastic analyses is that it supports elastic structures in the hydrodynamic analysis in the frequency domain, and it includes diffraction and radiation. Others have developed their own programs and coupled these together (Senjanovic et al., 2012), (Ishihara et al., 2007a). University of Southampton, in collaboration with Lloyd’s Register of London built a portal able to perform hydroelastic calculations in a cluster (University of Southampton, 2013), but it seems that this project have been cancelled (Temarel, 2013). The institute of Ship Science at University of Southampton are using the constituent programs from the portal individually.
2.4 Defining the degree of hydroelasticity

It is important to have knowledge about when a quasi-static analysis is sufficient and when a more advanced hydroelastic analysis must be performed when designing a floating structure. This is especially important for long and slender structures. Four methods that tries to define this limit are described below.

2.4.1 Global response

The degree of hydroelasticity may be determined with the use of a characteristic length according to Suzuki et al. (2006). The characteristic length is the length of which the deformed region becomes when a concentrated load is applied, as seen in figure 2.1.

\[ \lambda_c = 2\pi \left( \frac{EI}{k_c} \right)^{1/4} \]  

where \( k_c \) is the hydrostatic stiffness in heave which is equal to \( k_c = \rho g A_w \), \( A_w \) is the waterline area, \( \rho \) is the density of water, \( E \) is Young’s modulus for the material and \( I \) is the second moment of area. Conventional ships which are short in comparison to the characteristic length behave as a rigid body, shown in figure 2.1 (a). A VLFS on the other hand, that is long in comparison to the characteristic length behave as a flexible body, shown in figure 2.1 (b). In order to determine the global response of the structure the characteristic length is plotted against the length of the structure over the wave length, shown in figure 2.2.
If the length of the structure over the characteristic length is less than one, rigid body motions are dominating the total motion, else elastic motions are dominant. This ratio is plotted on the vertical axis of figure 2.2. If the elastic motions are dominant, hydroelasticity have to be taken into account.

Based on this reasoning, it is possible to perform a simplified analysis to provide an estimate of the degree of hydroelasticity for a structure. A semi-submersible, such as Hexicon’s structure, is not applicable to this method because the hydrostatic loading for a semi-submersible is not at all distributed as shown in figure 2.1, but rather point loads on the nodes. The definition of the characteristic length is thus not valid for semi-submersibles. Although, for a conventional ship or a VLFS the method is useful to get a first estimate on the degree of hydroelasticity.

2.4.2 Dynamics

Dynamics is the study of forces and their effect on motions. Within dynamics the equation of motion may be written as a mass-spring-damper system as

$$ F = a\ddot{x} + b\dot{x} + cx $$

(2.2)

where $a$, $b$ and $c$ is the mass, damping and stiffness of the system, respectively. $\ddot{x}$, $\dot{x}$ and $x$ represent the relative acceleration, velocity and position of the system, respectively.
An analogy may be drawn between the mass-spring-damper system and a floating structure. The spring stiffness corresponds to the hydrostatic stiffness in heave, the damping corresponds to the hydrodynamic damping and the mass of the system corresponds to the mass and added water mass of the structure. Subjected to a harmonic force, a dynamic system will also oscillate harmonically with the same frequency as the force, according to linear system theory. A harmonic transfer function of such a system can be seen in figure 2.3 for frequencies varying from zero to two times the eigenfrequency of the system. The loading frequency $\omega$ is equal to the wave frequency. The dynamic amplification factor, seen on the vertical axis of figure 2.3, is the transfer function normalized to unity using the stiffness of the structure. The damping of the system shifts the natural frequency to be slightly lower than the eigenfrequency for lightly damped systems, and this shift is described by

$$\omega_n = \omega_e \sqrt{1 - 2\xi^2} \quad (2.3)$$

where $\xi$ is the damping factor. $\xi$ is for lightly damped systems a few percent of the critical damping. It is observed in figure 2.3 that if the wave frequency is well below the natural frequency of the system (2.2), the response acts quasi-statically, else the system behaves dynamically and the flexibility of the structure must be incorporated in the analysis. A quasi-static loading condition may be assumed for wave frequencies up
to a limit of somewhere between a fifth and half of the natural frequency, highlighted in figure 2.3, depending on the accuracy needed. Seng et al. (2012) concluded that if the encounter frequency of a ship is close to the hull girder natural frequency, calculation methods based rigid body assumptions may not be accurately enough, which corresponds well with a dynamic system.

If the loading frequency is close to the natural frequency resonance will occur, as seen in figure 2.4 (a), and the amplitude is largely determined by the damping of the system. If the wet natural frequency is about 2-5 times the loading frequency, as seen in figure 2.4 (b), the response might be described as quasi-static.

The actual limit must be further investigated for global hydroelasticity. Considerations must be taken to an irregular wave spectra. The difficulties of an irregular spectra compared to a single wave is that an irregular spectra includes many frequencies of varying amplitude, not just one frequency.

2.4.3 Dynamic characterisation

Research on panel-water impact by Stenius et al. (2011) describes the dynamic characterization i.e. degree of hydroelasticity, of a panel located in the hull of a high speed craft. Their research concludes that if the loading time period is larger than two times the first wet structural natural period, the system can be assumed to be quasi-static. If it is not, the structure must be analysed hydroelastically.
Chapter 2. Degree of hydroelasticity

Using the conclusions from (Stenius et al., 2011) in a dynamic system, and converting time period to frequency as

\[ T_L > 2 \cdot T_e \rightarrow 2 \cdot \omega_L < \omega_e \]  \hspace{1cm} (2.4)

where \( T_L \) is the loading period, \( T_e \) is the first structural eigenperiod, \( \omega_L \) is the loading frequency and \( \omega_e \) is the eigenfrequency. The quasi-static assumption is valid when the wet natural frequency is higher than about two times the loading frequency. Stenius et al. (2011) models the local hydroelasticity with different boundary conditions than for a floating structure. Different boundary condition may yield different limits to when hydroelasticity needs to be included in an analysis. To be able to draw reliable conclusion for global a hydroelastic application a more detailed comparison must be performed.

2.5 Conclusions and discussion

The study of dynamics and of the dynamic characterization proposed by Stenius et al. (2011) concluded that if the structure’s first wet elastic natural frequency is higher than about 2-5 times the wave frequency the loading conditions may be assumed to be quasi-static. If the natural frequency is lower than this, hydroelasticity must be incorporated in the analysis. The actual limit must be investigated further to get a more reliable definition of global hydroelasticity, in regular and irregular waves.

The study of global response proposed by Suzuki et al. (2006) indicates if hydroelasticity is of concern for conventional ship or VLFS. For semi-submersibles this method is not applicable since the loading condition for these does not correspond to the loading condition assumed in the study.

Suggestions of future work is to analyse this limit more thoroughly. A more detailed model must be created to verify the limit for both regular and irregular waves. The most practical commercial software for this analysis is ANSYS with the use of their Morison-type elements if Morison’s equation is valid or Wamit coupled to a FE-model if diffraction theory must be included.
Chapter 3

Literature study: Floating Wind Turbines

3.1 Introduction

Onshore wind power has now been around for several decades, even offshore bottom-mounted wind turbines has been around for about 20 years, and in the recent years, the number of bottom-mounted offshore wind turbines has increased exponentially. In recent years concepts and prototypes of floating wind turbines have emerged that are not restricted to shallow water (depths less than about 40 meters), which the bottom-mounted are. These concept can instead be situated further out to sea where the wind is stronger and more persistent, and where the water is deeper. The deep parts of the North Sea, the Pacific Ocean and the Atlantic Ocean are areas where the potential for floating wind turbines is huge (EWEA, 2013). The wind energy potential along the U.S coast, up to 50 nautical miles from the shore, alone equals four times the yearly energy consumption of the United States, according to Breton and Moe (2009). 90 percent of the area of this energy potential have depths of more than 30 meters, hence the energy potential for floating wind turbines is huge.

3.2 Floating wind turbines

The different types of floating wind turbine platforms are all taken from the oil industry. Some of the most common structures are semi-submersible, pontoon, spar-buoy and tension leg platform, as figure 3.1 illustrates. The semi-submersible first appeared by mistake more than 50 years ago in the oil industry. And since then, much progress has
been made and other types of platforms have emerged such as the spar buoy and tension leg platform.

![Figure 3.1: From left: Semi-submersible, Ponton, Spar-buoy and TLP](image)

**Semi-submersible** is a type of floating structure, which indicate that a large part of the buoyant body is located below the surface with only a small waterline area. This differs from ordinary vessels where the buoyant body is located close to the surface and it has got a large waterline area. The change in the hydrostatic force is proportional to the waterline area, which means that the influence of waves on a semi-submersible is less than of ordinary ships.

**Pontoon**s have, as ordinary ships, all its buoyancy located about the waterline and due to this large waterline area they are very sensitive to waves, and more prone to move with the waves.

**Spar buoys** have also got a small waterline area in order to minimize the change in the hydrostatic force in the interaction with waves. They have a very deep draught and their centre of gravity is located far below the water surface and are thus very stable in heave and pitch.

**TLP** are platforms that are held in place by vertical wires that are moored to the bottom. The wires are pre-tensioned which means that they pull the platform down a small distance below the equilibrium floating position. These cables have very high axial stiffness and together with a small waterline area and the pre-tensioned wires the motions of the platform in heave are very small.

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13Tension Leg Platform
3.3 Background on other wind turbines

Today there are only a few floating wind turbine prototypes at full scale in operation, two of which are Hywind and WindFloat. Hywind is a spar-buoy, developed by Statoil and it was the first floating wind turbine in the world to be tested in full scale (Wikipedia, 2013b). It is located off the coast of Norway since 2009. WindFloat is the semi-submersible type floater and it is being tested off the coast of Portugal since 2011. Both of these floating wind turbine platform have been widely used for research in this field and to develop standards for classification society e.g. DNV (DNV-OS-J103, 2013). Significantly more floating wind turbine projects are in the design stage and several have also completed tank-testing, most of them in Japan or Europe (LLC, 2013), (EWEA, 2013). Off the coast of Fukushima, Japan, there is a full-size prototype floating wind turbine installed and Japan plans to expand this by building two more full scale prototypes (Fukushima-Forward, 2013). The WindFloat turbine, the Hywind turbine, the Hexicon concept and Ishihara’s concept is shown in figure 3.2.

![Image of floating wind turbines](image)

**Figure 3.2:** Illustration of some concepts of floating wind turbines. From left: WindFloat, Hywind, Ishihara’s concept and Hexicon

Some areas of considerations when designing a floating wind turbine are the wind and wave forces, the structure and the mooring. Different scientists have chosen different modelling techniques for their unique floating wind turbine concept. Here follows a short summary of a few concepts and the researchers respective modelling technique.
The fluid-structure interaction simulations can be done either in time or frequency domain. Calculations in the frequency domain has the advantage of being more time-efficient and easy to perform, but in the time domain non-linear effects such as slamming and viscous drag can be included (Taghipour, 2008). The time domain simulations are also more intuitive to understand because you get motions in a time series instead as a function of frequency. (Taghipour, 2008) has made calculations of both the hydrodynamic coefficients and motion calculations in the frequency domain while (Kvittem et al., 2012) & (Ishihara and van Phuc, 2007b) calculated the hydrodynamic coefficients in the frequency domain, transferring the results and calculated the response in the time domain. (Ishihara and van Phuc, 2007b) made simulation in time domain. (Kvittem et al., 2012) compared different hydrodynamic theories for determination of the dynamic response. Potential theory, in which diffraction is included, is compared with various configurations of Morison’s equation, where the wave force is integrated to the mean water level and where the wave force is integrated up to the instant wave elevation. (Taghipour, 2008) mentions a hybrid time-frequency method to include non-linear forces in the frequency domain where the solution from the frequency domain is transferred to the time domain where these forces are included.

The structure is modelled as rigid by both (Jonkman, 2010) and (Kvittem et al., 2012) while (Ishihara and van Phuc, 2007b) compares a rigid and an elastic model and reaches the conclusion that an elastic response occurs when the wave frequency is close to the wet elastic natural frequency of the structure.

The mooring is modelled as linear springs in (Ishihara et al., 2007a) while in a later report they modelled the mooring non-linear (Ishihara and Van Phuc, 2007b). (Kvittem et al., 2012) have modelled the mooring as non-linear. (Jonkman, 2010) compares a linear and non-linear mooring and suggest that a linear mooring approximation is sufficient if the relative motions are small.

Wave theory chosen by most of the authors is the linear wave theory, while (Ishihara et al., 2007a) compared the linear and non-linear wave theory (the stream function up to the 9:th order) and arrived at the conclusion that non-linear effects are relevant when the water depth over wavelength is less than 0.5, which also is the common definition of deep water. The author also conclude that elastic modes may become in resonance with the higher order harmonic components from the non-linear waves.
3.4 Design levels

DNV have with the release of their offshore standard ‘Design of floating wind turbine structures’ (DNV-OS-J103, 2013) defined different design levels suitable for different stages in the design process a structure is in, corresponding to different level of detail. These design levels concerns five different categories, which are floater, tower, hydrodynamic loads, aerodynamic loads and mooring. Since this thesis doesn’t focus on the tower or aerodynamics, this has been left out but can be seen in (DNV-OS-J103, 2013). The floater, hydrodynamic loads and the mooring from (DNV-OS-J103, 2013) are presented in table 3.1. These categories all consist of three levels; level 1 is the least advanced level of analysis, level 2 is the intermediately advanced level of analysis and level 3 is the most advanced level of analysis.

Table 3.1: DNV design levels.

<table>
<thead>
<tr>
<th>Category</th>
<th>Level 3</th>
<th>Level 2</th>
<th>Level 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>The floater</td>
<td>-Correct mass and stiffness properties</td>
<td>-Some simplifications of the floater may be used (e.g. the use of concentrated masses, non-linear contributions disregarded)</td>
<td>-Simplified as a mass-spring-damper system</td>
</tr>
<tr>
<td></td>
<td>-Other aspects relating to the floater type shall be taken into account</td>
<td></td>
<td>-Estimating the main properties to resemble the actual structure</td>
</tr>
<tr>
<td>Hydrodynamic</td>
<td>-Diffraction &amp; radiation taken into account.</td>
<td>-As level 3, but the use of Morison’s equation accepted if it’s considered suitable</td>
<td>-Loads may be analysed given time series or look-up tables</td>
</tr>
<tr>
<td>loads</td>
<td></td>
<td>-Drag and inertia coefficients may be constant over time</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Non linear contributions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-For simulations in time domain: added mass and damping shall be determined for a range of frequencies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mooring</td>
<td>Catenary:</td>
<td>Catenary:</td>
<td>-May be calculated separately</td>
</tr>
<tr>
<td></td>
<td>-The real dynamics of the mooring lines reproduced</td>
<td>-A quasi-static model may be used</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Buoyancy and drag of mooring lines included TLP:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Soil conditions contribute to damping</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Synthetic ropes exhibits non-linear deformation and should be included</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
These design levels very much resemble the conceptual, preliminary and detailed design stages proposed by Keane et al. (1986) in their analysis of three SWATHs\(^{15}\).

Worth noting is that DNV does not account for structural dynamics for the floater. For slender structures, where hydroelasticity might be of concern, structural dynamics must be incorporated in an analysis, else the stresses and motions will be underestimated where resonance occur.

At design level 3 for the floater: hydroelasticity should be included if it is relevant to the concept, i.e. if the first wet elastic natural frequency is lower than 2-5 times the wave frequency in regular waves or roughly five times the peak frequency in irregular waves. At design level 2 for the floater: hydroelasticity should be included in a simplified manner if relevant to the concept. At design level 1 for the floater: hydroelasticity does not need to be included but the knowledge of if it is relevant to the concept should be known.

### 3.4.1 Design level 1 - the least detailed design level

A short description on how the analysis may be in the least detailed design level, design level 1, is here described. This design level corresponds to a conceptual design stage, which would answer to if the concept is possible to realize.

**The hydrodynamic forces** may be defined using a quasi-static analysis of the waves, neglecting the dynamic wave force components.

**The floater** may be defined as a rough sketch of the structure, neglecting a lot of detail.

**Hydroelasticity** may be neglected, even for slender structures. A simple free vibration analysis should be performed, including a simple estimation of the added mass, to get an estimation of if hydroelasticity must be taken into account according to chapter 2.

**The mooring** may be calculated separately and included afterwards.

\(^{15}\)Small Waterplane Area Twin Hulls
3.4.2 Design level 2 - the intermediate detailed design level

A short description on how the analysis may be performed in the intermediate detailed design level, design level 2, is here described. This design level corresponds to a preliminary design stage, which would answer to if the stresses that arises from the applied loads are within the chosen margin safety, using preliminary dimensions of the structural parts.

The hydrodynamic forces may be defined using Morison’s equation and neglecting diffraction if Morison’s equation is valid for the analysed waves. Otherwise the more advanced potential flow including diffraction method must be used. The drag term in Morison’s equation may be linearised for analyses in the frequency domain. The waves may be defined as linear waves and an irregular sea spectra may be used. Non-linear waves and slamming may be disregarded. Drag and inertia coefficients should depend on the Keulegan-Carpenter number.

The floater may be defined with the most important structural parts, neglecting only smaller details.

Hydroelasticity must be included if the structural elements are slender and if the natural frequency is close to the sea spectra, as described in chapter 2. If this is not the case a rigid body assumption is sufficient.

The mooring may be modelled as linear quasi-static springs in the surge and heave direction, neglecting the damping due to the soil conditions for TLPs and the viscous drag and mass term for catenary moorings.

3.4.3 Design level 3 - the most detailed design level

A short description on how the analysis may be in the most detailed design level, design level 3, is here described. This design level would correspond to a detailed design stage, which yield the final dimensions of the entire structure and serve as material for a blueprint.

The hydrodynamic forces may be defined using potential flow theory taking diffraction and radiation into account and calculating the Keulegan-Carpenter number dependent i.e. frequency dependent, added mass and drag coefficients. The analysis may be
performed in the frequency domain to save time and CPU-power. A more sophisticated CFD-method may be used to analyse extreme events e.g. slamming, monster waves and green water on deck. Time domain simulations are performed for e.g. the largest wave during the return period, at sea states where high stresses or motions occur and at the extreme events. Where the time domain simulations are performed on the largest wave during the return period, Stoke’s waves may be used, if it’s considered suitable. Table A.4 in (DNV-OS-J103, 2013) describes floater-specific issues such as the length of the time simulation. It states that time domain simulations must have a length minimum of three hours.

The floater must be modelled with all subcomponent in order to obtain results that corresponds well with the real structure.

Hydroelasticity must be included if the structural elements are slender and if the natural frequency is close to the sea spectra, as described in chapter 2. If this is not the case a rigid body assumption is sufficient.

The mooring may be assumed to be of TLP or catenary type. A catenary type mooring is non-linear in nature and must hence be modelled non-linear to be able to account for large motions that might occur, e.g. large surge amplitudes. The complete dynamics of the mooring must be taken into account, including the mass and damping of the mooring lines. Also drag and buoyancy forces from the mooring lines must be included in the analysis. For TLP type moorings damping due to certain soil conditions must be investigated and implemented. If synthetic ropes are used, the non-linear elastic behaviour of these ropes must be implemented in the analysis.

3.4.4 Corresponding design levels

The corresponding design levels for the authors in the previous chapter are here presented. Both (Jonkman, 2010) and (Kvittem et al., 2012) are considered to be close to, but does not reach, design level 3. They both uses advanced modelling methods for determining the hydrodynamic loads and include great detail of the structure. The reason that they don’t reach design level 3 is that none of the above does take into account extreme events such as slamming or monster waves and that (Jonkman, 2010) simplifies the mooring by e.g. neglects the hydrodynamic damping, and (Kvittem et al.,
2012) only mentions a certain software for computing the mooring forces, by which it is difficult to determine the level of detail in their analysis. These analyses are considered to comply with the requirement for design level 2. (Ishihara et al., 2007a) is also considered to comply with the requirements for design level 2. They use Morison’s equation to calculate the hydrodynamic loads and model the mooring using linear springs. The floater is built up using beam element with the main structural elements only, and a small amount of detail.

### 3.5 The study from GVA

The hydrodynamic model created by GVA of Hexicon’s platform containing 24 wind turbines is a semi-flexible analysis of the platform. It consists of several rigid bodies that are interconnected with each other using linear springs. The division of the platform into several rigid bodies is an attempt to recreate a flexible structure with the use of rigid bodies. This is used since the wave analysis software that GVA uses (WADAM\(^{16}\)) doesn’t take into account elastic structures, it assumes rigid bodies. Figure 3.3 shows a schematic illustration of how this performed.

\[\text{Figure 3.3: The centre node to the left and an outside node to the right, interconnected with springs.}\]

In an attempt to get around this, they did divide the large body into five individual bodies and connected them together using translative springs in the x-, y-, and z-direction. The spring’s stiffness’, shown in table 3.2, were chosen to be large to resemble a stiff

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\(^{16}\)WADAM is a wave analysis program developed by DNV which is based on the theory of earlier versions of WAMIT
connection in these directions. A torsional spring in the $\theta$-direction is also included in the table, but with zero stiffness.

<table>
<thead>
<tr>
<th>Spring stiffness'</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_x$</td>
<td>10000</td>
</tr>
<tr>
<td>$k_y$</td>
<td>10000</td>
</tr>
<tr>
<td>$k_z$</td>
<td>10000</td>
</tr>
<tr>
<td>$k_\theta$</td>
<td>0</td>
</tr>
</tbody>
</table>

This means that, at the interconnection point, there is only translative stiffness in all directions, and no torsional stiffness. When such a structure is excited by waves, the individual bodies will move almost independently of each other and not as a single structure as a flexible structure does. Pitch of the individual bodies is mentioned in the study, which would sort of correspond to a flexible rotation for the actual model, but the springs are not designed to correspond to the rotational angle of a flexible structure. Heave for the individual bodies is also mentioned in the study, but again, it has not been designed to correlate to a flexible structure. One could sort of correspond heave to an elastic deflection in each node of the actual model. The only reason to apply translative springs is that the individual bodies doesn’t float away from each other in the interaction with waves and to obtain a flexible interconnection. All in all, this model has large potential for improvements.

### 3.5.1 Design level of GVA’s report

The amount of details in the structure is rather large, but it’s considered to be according to design level 2. The hydrodynamic loads are also according to design level 2, hence the poor inclusion of hydroelasticity for such a large structure and there that no extreme events are modelled. The mooring is only mentioned as a stiffness in the report and is thus considered to be according to design level 2. Over all the report is considered to analyse the structure according to design level 2.

### 3.5.2 Improvements

With the insufficient inclusion of hydroelasticity there is a large potential for improvements. The idea is here to design the torsional spring $k_\theta$ so that it resembles an elastic
beam that is clamped to the node. The clamped condition is obtained by giving the three translative springs infinite stiffness. The torsional spring stiffness is designed so that the deflection at the tip of the hinged beam is the same as the deflection of a cantilever beam, where the beam attached by torsional springs is denoted hinged beam. Both of these beams are of the same length, $L$. The deflection at the end for the cantilever beam is $\delta_{\text{elastic}}$ and the for the hinged beam is $\delta_{\text{hinge}}$, which are determined below, respectively, when a static load $P$ is applied at the end. $EI$ is the stiffness of the cantilever beam.

$$\delta_{\text{elastic}} = \frac{PL^3}{3EI} \quad (3.1)$$

$$\delta_{\text{hinge}} = \frac{PL}{k_\theta} \quad (3.2)$$

The deflection at the end should be the same for both beams, which means that

$$\frac{\delta_{\text{elastic}}}{\delta_{\text{hinge}}} = \frac{3EI}{k_\theta L^2} = 1 \quad (3.3)$$

must be true. The torsional stiffness of the hinged beam is thus

$$k_\theta = \frac{3EI}{L^2} \quad (3.4)$$

For the hinged beam, the deflection is exact at the end of the beam but nowhere else, as seen in figure 3.4.

![Figure 3.4: Comparison between an elastic beam and a hinged beam, divided into one respective two parts to equal the elastic beam.](image)

If the beam is split up into several smaller beams interconnected with each other using torsional springs the deflection will approach the exact for more and more element
divisions. Figure 3.4 also shows two hinged beams with a deflection that corresponds to the elastic beam. The beams are a hinged beam with one element, a hinged beam divided in two equally long parts and an elastic beam. A problem with this procedure is that the particular wave analysis software can only handle up to 15 separate bodies, which might be too few bodies to describe the structure sufficiently accurate. Bending moments and shear stresses may be determined by differentiating the deflection twice, respectively three times but it would require many more than two element per beam for the stresses to be reliably predicted. A convergence analysis must be performed to determine the number of element division required to obtain reliable results.

The vertical deflection at the end of the beam is the same for all beam divisions, but the angle (pitch) is different. For the hinged beam the angle at the end tends to the angle of the elastic beam for $N \to \infty$ number of divisions.

3.6 Conclusions and discussion

Global hydroelasticity is not accounted for in many cases, however it is not a problem for non-slender structures, which most of the floating wind turbines probably are. When designing a floating wind turbine, or any other floating structure for that matter, knowledge of if hydroelasticity is relevant to the concept should be known.

Global hydroelasticity is not included in the DNV design levels, not even mentionend, which is a big shortcoming from their side. The semi-flexible model used by GVA in the report analysing one of Hexicon’s platform was found to be insufficient and didn’t capture the hydroelastic behaviour correctly. A better model must be created to account for hydroelasticity in a correct manner.
Chapter 4

Simplified hydroelastic analysis of the Hexicon platform

4.1 Introduction

A simple fluid-structure interaction model is here described, including hydroelasticity, to get the feeling of the hydroelastic behaviour of one of Hexicon’s platforms. Idealisations of the structure has been carried out to get a manageable working load within the frames of this thesis, but still resemble the real structure as good as possible. The model is set up to acquire deformations, stresses and the degree of hydroelasticity for a two-dimensional beam consisting of a beam connected to two nodes (figure 4.1 (b)), similar to Hexicon’s structure which has several beams connected to several nodes (figure 4.1 (a)). Two beams with different properties are analysed, the first beam is designed so that the first wet elastic natural frequency is outside the sea spectra and the second beam is designed so that this frequency is almost within the sea spectra.

4.2 Idealisations

Idealisations are made to simplify the analysis, to reduce the modelling time and to remove parts less important for the model. Hexicon’s structure, seen in figure 4.1 (a), consist of several nodes where wind turbines are placed upon, one node in the middle with mooring connections and beams of various lengths connecting the nodes. The beams are built up with top and bottom beams and diagonal braces in between.

This thesis analyses two nodes connected to each other using a beam. The beam in the model is of the same stiffness as the beams between the nodes in Hexicon’s platform but modelled as a single, one dimensional Euler beam. The weight of the towers are
included in the model as masses on top of the nodes. As can be seen in figure 4.1 (a) the height of the beam in Hexicon’s platform is the same height as the node and the stiffness for the node and the beam is in the same order of magnitude, the node could thus be seen to be a part of the beam. The two-dimensional model is seen in figure 4.1 (b). These simplifications are done in order to simplify the analysis and still be able to account for hydroelasticity using linear hydroelasticity theory.
A free-free boundary condition is assumed, which is a common boundary condition for floating structures and according to the procedure in (Bishop and Price, 1979).

4.3 Assumptions

Based on the review of some of the other floating wind turbine concepts and reasoning about how much that can be done in the duration of the thesis, the assumptions made and modelling technique chosen in this thesis are:

- Long crested waves are assumed; so that Airy wave theory may be used. Airy theory (linear wave theory) is the simplest wave theory and also the most used because of its simplicity and its ability to produce irregular waves with.

- Small bending deformations are assumed; so that linear beam theory may be used.

- The beams are modelled as one-dimensional Euler beams with shear deformation and rotary inertia neglected, which is a reasonable assumption for thin beams.

- Deep water is assumed; To simplify the velocity potential equation (and because the platform is supposed to be situated in deep water for most of waves).

- The wave forces are assumed to correspond to Morison’s equation.

- The added mass has been assumed to be equal the displaced volume times the density of the water, which is equal to the theoretical value for potential flow.

- The wave forces are only modelled on the nodes and neglected on the beams.

- The nodes including towers and nacelles are modelled as point masses on the first and last beam element.

- A free-free boundary condition is assumed, which is a common boundary condition for structures in waves and according to (Bishop and Price, 1979).
4.4 Main data

The main dimensions of the Hexicon platform can be seen in figure 4.2 and the main dimensions of the two-dimensional model can be seen in figure 4.1 (b). The most important dimensions are summarized in table 4.1. Two different beams have been subjected to analysis, beam no. 1 and beam no. 2. The difference between those two beams are the lengths and second moment of inertia, seen in table 4.1. The beams cross section resembles the Hexicon beams, i.e. top and bottom beams that take up the bending stress and diagonal braces in between which take up the shear stress.

![Diagram of Hexicon platform](image)

**Figure 4.2:** Side view of the Hexicon platform, showing the stillwater level.

### Table 4.1: Main dimensions for the beams and the Hexicon platform

<table>
<thead>
<tr>
<th>2D model</th>
<th>beam no.1</th>
<th>beam no.2</th>
<th>symbol</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>459</td>
<td>506</td>
<td>$L$</td>
<td>m</td>
</tr>
<tr>
<td>Draught</td>
<td>17</td>
<td>17</td>
<td>$T$</td>
<td>m</td>
</tr>
<tr>
<td>Diameter of node</td>
<td>10</td>
<td>10</td>
<td>$D$</td>
<td>m</td>
</tr>
<tr>
<td>Neutral axis of the beam</td>
<td>$-4$</td>
<td>$-4$</td>
<td>$z_{NA}$</td>
<td>m</td>
</tr>
<tr>
<td>Total mass</td>
<td>21450</td>
<td>25840</td>
<td>$m$</td>
<td>tonnes</td>
</tr>
<tr>
<td>Second moment of inertia</td>
<td>96</td>
<td>48</td>
<td>$I$</td>
<td>$m^4$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>200</td>
<td>200</td>
<td>$E$</td>
<td>GPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hexicon’s platform</th>
<th>Hexicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>378</td>
</tr>
<tr>
<td>Breadth</td>
<td>506</td>
</tr>
<tr>
<td>Draught</td>
<td>13</td>
</tr>
<tr>
<td>Diameter of node</td>
<td>10</td>
</tr>
<tr>
<td>Beam height</td>
<td>26</td>
</tr>
<tr>
<td>Mass</td>
<td>27424</td>
</tr>
<tr>
<td>Second moment of inertia of one beam</td>
<td>48</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>200</td>
</tr>
</tbody>
</table>


A close up on the left node.

**Figure 4.3:** Side view of the left node. $z_{NA}$ is the distance from the stillwater level to the neutral axis of the beam, $T$ is the draught and $D$ is the diameter of the node.

### 4.5 The model

The modelling of the fluid-structure interaction between the waves and the structure is here presented. A hydrodynamic model is made to simulate the wave forces on the nodes and a structural model that calculates the deformation, bending stress and shear stress of the beams that arise from the wave loading. The hydrodynamic and structural model is coupled to capture hydroelastic effect that may occur.

#### 4.5.1 The chosen design level

The chosen level of detail in the modelling complies with the requirements for design level 2.

**The floater and hydrodynamic forces** are modelled according to the intermediate design level, design level 2. Morison’s equation is used to calculate the forces on the structure. This is approximately valid for the ratio of wave lengths over structural diameter larger than 1/7. The force on the beams in between the nodes are neglected, although the added mass is included to resemble the correct natural frequency. Non-linear effects such as slamming and non-linear waves are disregarded, although the drag force is linearised so that it may be included. Drag and inertia coefficients are constant with respect to the Keulegan-Carpenter number. Other aspects related to the floater type e.g. slamming, added mass on bracings, have been disregarded.
Hydroelasticity is not account for in DNV, but it is done herein, and it is here considered to correspond to design level 2.

The mooring is modelled according to design level 2; as a linear, massless, spring in the heave direction.

### 4.5.2 Hydrodynamic model

The hydrodynamic model is a linearised Morison’s equation to calculate the wave loads where diffraction has been neglected. Linear airy wave theory is used to model the waves, which is beneficial since it’s the most simple wave model, yet accurate for long crested waves i.e. small wave height over wave length ratio, very easy to implement and it’s possible to model either regular or irregular waves. The wave force equation is constructed as follows

\[ F_{\text{wave}} = F_{\text{Morison}} + F_\zeta \]  \hspace{1cm} (4.1)

where \( F_{\text{Morison}} \) is Morison’s equation and \( F_\zeta \) is the vertical stiffness term. The wave forces are modelled on the nodes but neglected on the beams to simplify the loading condition. Inclusion of wave forces on the beam would probably yield larger heave response but not necessarily larger elastic responses. The added mass is modelled on the beams to obtain the correct natural frequencies. The wave forces are modelled as dynamic with an inertia-, hydromechanical damping- and hydromechanical stiffness term, which are proportional to the water particle acceleration, water particle velocity and effective wave amplitude, respectively. The effective wave amplitude is the effect that the wave amplitude has on a structure at a certain depth. The acceleration and velocity terms may be differentiated to be linear in relation to the effective wave amplitude.

The added mass for large frequencies for a circular two-dimensional cylinder is equivalent to the displaced volume of the cylinder times the density of water (DNV-RP-C205, 2010). In waves, the added mass changes depending on the frequency of the waves. This frequency dependence is in offshore engineering commonly related to the dimensionless parameter Keulegan-Carpenter number, i.e. period number. Keulegan-Carpenter number is the ratio of the wave particle velocity, \( u \), times wave period, \( T_w \), over the diameter, \( D \).

\[ K_C = \frac{uT_w}{D} \]  \hspace{1cm} (4.2)
The frequency dependence is herein assumed to be constant with respect to the wave frequency. The inertia term is called the Froude-Krylov force term (4.3) and is equal to the force on a structure induced by the pressure of an undisturbed wave at the position of the structure, as if the structure wasn’t there.

\[ F_{FK} = C_a \rho V \dot{u} \] (4.3)

where \( \dot{u} \) is the wave particle acceleration, \( C_a \) is the added mass coefficient and \( V \) is the volume of the submerged body. The damping term i.e. drag term, is the force component that is proportional to the water particle velocity squared. For a cylinder, the drag term per unit length is

\[ F_D = \frac{1}{2} C_d \rho A_{ref} u^2 \] (4.4)

where \( A_{ref} \) is the reference area, \( \rho \) is the density of water, \( u \) is the water particle velocity and \( C_d \) is the drag coefficient. The drag coefficient is assumed to be equal to 1 according to (DNV-RP-C205, 2010). When combining the inertia term and the drag term Morison’s equation is obtained, see equation (4.5). It is commonly used in the offshore industry for determining loads on slender structural elements in waves.

\[ F_{Morison} = F_{FK} + F_D = C_a \rho V \dot{u} + \frac{1}{2} C_d \rho A_{ref} u^2 \] (4.5)

Morison’s equation was developed to determine the wave loads on slender cylindrical structures, which is exactly what the horizontal forces are in this case. The vertical forces on the node ends does not correspond to this, but the added mass term in the Froude-Krylov equation may be altered to account for pile endings as done in (Haslum and Faltinsen, 1999).

\[ F_{FK} = \rho C_a \frac{2\pi}{3} \left( \frac{D}{2} \right)^3 \dot{u}_z \] (4.6)

An investigation, found in appendix A.1, concludes that Morison’s equation is valid for structures with a diameter over wave length that is smaller than 1/7 and that diffraction may be neglected in this case. In the case of a 10 meter wide node this corresponds to 70 meter long waves.

To be able to include the drag term in a frequency domain analysis, as done herein, it must be linearised. In this frequency domain analysis this force component must be
linear with respect to the water particle velocity. This linearization process is briefly described in appendix A.5 and the linearised drag force is

$$F_{Dlinear}^{linear} = \frac{1}{2} C_d \rho A_{ref} \sqrt{\frac{8}{\pi}} \sigma_u u$$  \hspace{1cm} (4.7)$$

where $\sigma_u$ is the standard deviation of the water particle velocity of a given irregular sea response spectra.

Morison’s equation was developed for fixed, bottom mounted structures, and does calculate the dynamic wave force on the structure. The vertical stiffness term is added to Morison’s equation to capture the varying buoyancy of a freely floating structure in waves, which gives rise to vertical motions. A fixed, bottom mounted structure is restrained from vertical motions and this term doesn’t appear there. It term can be seen as a Froude-Krylov term in calm water, where the wave particle velocity and acceleration is equal to zero, and it is proportional to the location of the wave, or the instant wave height $\zeta$.

$$F_{\zeta} = \rho A_w \zeta$$  \hspace{1cm} (4.8)$$

The horizontal stiffness term is only comprising of the mooring stiffness in the horizontal direction.

A more detailed description of the wave loads are given in appendix A.3.1.

4.5.3 Waves

Waves created by the wind are irregular by nature and therefore it is not sufficient to only study regular waves. Irregular waves are built up by superimposing regular waves with different amplitude, frequency and phase. They are defined by a sea spectra describing the energy density of the various regular wave components included in the irregular waves.

A sea spectra of a typical storm is shown in figure 4.4 where the significant wave height, $H_S$, is 8 meters and the peak period, $T_P$, is 12 seconds. It can here be seen that the density intensity of the waves in this spectra is largely focused on the frequencies in between 0.4 – 1.0 rad/s. At this sea spectra, with the largest wave have a frequency of about 0.5 rad/s which corresponds to a wave length of about 250 meters. The deep
water assumption is valid for water depths of 125 meters and deeper, which is not at all an unlikely water depth for the platform. Although for larger waves the deep water assumption starts being questionable and leads to an under estimation of the wave loading.

For a more detailed description of the wave theory, consult appendix A.2.

### 4.5.4 Structural model

The structural model considers structural dynamics of the beam i.e. dynamic excitation and response, using a modal method. The beams are built up by a FE-model consisting of Euler beam elements. Each small element is \( dx \) long and there are \( J \) number of elements and is schematically illustrated in figure 4.5. The deformation of element \( j \) depends on the deformation of element \( j+1 \) and \( j-1 \). The beam is assumed to have a free-free boundary condition, which is according to the procedure done in (Bishop and Price, 1979).

The elements are idealised as mass-spring-damper both in vertical translation and in rotation for the respective beam elements. All the elements are of the same stiffness, mass and damping properties. The nodes, including the weight of the towers and nacelles, are modelled as point masses on the first and last element. The global d.o.f. consists of heave, pitch and the elastic d.o.f. (rigid body motions and elastic motions) and the local
Figure 4.5: The idealisation of the beam as small Euler beam elements. (a) represent the vertical spring and (b) represent the vertical damper.

d.o.f. corresponds to the number of small beam elements \( j \) that the beam is divided into.

Also the structural forces are idealised as a dynamic with an inertia, structural damping and structural stiffness term proportional to the elastic acceleration, elastic velocity and relative position of the beam, respectively. The inertia term is the total mass of the structure combined with the added mass of the structure. The structural damping is generally difficult to determine. It depends on the chosen material, number and types of joints, internal and external fluids and more (Prof. Anderson, 2013). The damping factor is determined in experiments for the certain structure but an approximation is to give the damping factor a value of 3-7% of the critical damping in this case, which is a typical value for metal structures with joints (Irvine, 2004). Using this damping factor the structure may be defined as a lightly damped structure. If the wave forces were modelled on the beams the damping would be higher than 3-7% with all the surrounding water. The structural stiffness is the bending stiffness of the beam i.e. Young’s modulus times the second moment of area of the beam.

A more detailed description of the structural forces are given in appendix A.3.2.

4.6 Linear hydroelasticity

The linear hydroelasticity theory is combining both the hydrodynamical and structural models in the fluid-structure interaction analysis. (Bishop and Price, 1979) model both the hydrodynamics and structure using dynamic models, and by doing this the different models are easily combined. It is even possible to solve it analytically for very simple geometries. As described in chapter 2, hydroelasticity is of concern if the first elastic natural frequency of the structure is less than about 2-5 times the loading frequency.
The sum of forces are combined using Newton’s second law

\[F^{\text{wave}}(\dot{u}, u, \zeta) + F^{\text{rigid}}(\ddot{z}, \dot{z}, z) + F^{\text{elast}}(\ddot{z}, \dot{z}, z) + F^{\text{moor}}(z) = 0,\]  

(4.9)

where \(F^{\text{wave}}(\dot{u}, u, \zeta)\) are the wave forces on the structure, \(F^{\text{rigid}}(\ddot{z}, \dot{z}, z)\) are the forces due to rigid body motions, \(F^{\text{rigid}}(\ddot{z}, \dot{z}, z)\) are the forces due to the elastic motions, \(F^{\text{moor}}(z)\) is the mooring force, \(\dot{u}, u, \zeta\) is the wave particle acceleration, wave particle velocity and effective wave amplitude, respectively and \(\ddot{z}, \dot{z}, z\) is the acceleration, velocity and relative displacement of the structure, respectively. The response calculated is the deformation over wave amplitude, and the analysis is performed in the frequency domain. The upside of performing calculations in the frequency domain comparing to the time domain is that it is time efficient and modelling is simple using a modal analysis. The downside is that non-linear contributions must be linearised in order to be implemented. Structural dynamics concerns the dynamic behaviour of structures, in contrast to the static behaviour. All structures has got frequencies where the structure will experience resonance, and these frequencies and resonance amplitudes are important to have knowledge about. An efficient method of finding these frequencies and amplitude is using a modal method. A modal method consists of:

- finding the mode shapes and its corresponding natural frequencies,
- calculating the response of each mode for a given external load at all frequencies and
- superimposing the modal responses to a total response of the structure at all frequencies.

The natural frequency is the frequency of which the structure resonate at, and the mode shape is the shape the structure has at that particular frequency, shown in figure 4.6. This method is very time efficient and it is possible to find analytical solutions for simple structures such as one-dimensional, uniform beams. The bending moment and shear forces are derived from the deformation of the beam, which is obtained in the modal method.

For a more detailed description of the modelling method, consult appendix A.4.
Figure 4.6: Mode shapes of mode 0 through 3. Mode 0 and 1 corresponds to rigid body modes (heave and pitch) and modes higher than, or equal to two are elastic modes. All of the modes are normalized to unity.


4.7 Results

Results presented here are deformations, maximum bending stresses and maximum averaged shear stresses for the two beams.

4.7.1 Deformations

The deformations are here presented as elastic RAOs\textsuperscript{17} for beam no. 1, figure 4.7, and for beam no. 2, figure 4.8. These deformations are shown for frequencies varying from 0 – 2 rad/s and over the entire length of the beam.

\textsuperscript{17}Response Amplitude Operator
For beam no. 1, figure 4.7, the response for low frequencies show only small elastic behaviour, but for frequencies closer to the first elastic natural frequency the response show large dynamic behaviour. At the first elastic natural frequency, 1.99 rad/s, the largest deformation of about 1.5 meters is observed at both ends for wave amplitudes of 1 meter.

![Displacement as a function of position and frequency, L = 459 m.](image)

**Figure 4.7:** Elastic RAO for beam no.1

For beam no. 2, figure 4.8, the response show a large dynamic behaviour. This is because the first elastic natural frequency is 1.14 rad/s which is much lower than for the other beam. The largest deformations are observed at this frequency and are almost 2.5 meter at both ends for wave amplitudes of 1 meter.

These largest amplitude that occurs around the natural frequency of the beam are largely determined by the damping of the system. The damping of the system is determined by the drag coefficient and the structural damping. These properties are a very uncertain and small changes of these will give rise to large changes in the amplitude in the vicinity of the natural frequencies. Hence, these largest amplitude should not be seen as exact values but as preliminary deformation levels. More knowledge about the drag coefficient must be obtain in order to be able to determine the damping of the structure. The drag coefficient must be determined in experiments. Where these deformations occur is of great concern. If they occur within the sea spectra of interest resonance and large amplitudes will occur. Increasing the stiffness ($EI$), reducing the weight including added
mass \((m)\) or reducing the length \((L)\) of the beam will attain these results, as the wet elastic eigenfrequencies are defined as

\[
\omega_{e,n} = (\beta_n L)^2 \sqrt{\frac{EI}{mL^3}}
\]  

(4.10)

where \(\beta_n\) is a coefficient that is constant for the \(n\):th eigenfrequency. Installing heave plates\(^{18}\) at the bottom of each node will increase the viscous damping of the structure and thus reduce the vertical amplitude of both rigid body motions and elastic motions, especially in the vicinity of the natural frequencies. However, heave plates increases the added mass, which in turn lowers the natural frequencies.

If the towers and nacelles was modelled the natural frequency might alter a bit. The model with the given idealisations resemble reality in an adequate way at wave lengths in about the same length as the beam, but when several wave lengths equal the beam length, i.e. at higher frequencies, does the model probably over estimate the deformations. A more accurate model would be to model the wave forces on the beams as well as on the nodes.

Around the resonance frequency, a question of whether large bending deformations may or may not occur is raised. A general rule-of-thumb regarding small deformations is that the deformations should be smaller than \(1/10\) of the beam height. With the beam

\(^{18}\)Large plates that makes vortex shedding occur and thus increases the vertical drag force significantly.
height being about 26 meters high, large bending would occur if the deformations are larger than 2.6 meters. Using this rule-of-thumb, both beams are considered to have small deformations but beam no. 2 is right on the limit.

Two other rule-of-thumb considering when bending deformations are small are:

- deformations smaller than 1/20 of the largest beam dimension (SolidWorks, 2010)
- bending curvatures is larger than ten beam heights, (Wikipedia, 2014).

The results vary quite a lot depending on which ratio is chosen. Using the two last mentioned ratios to define small bending deformations, the deformations of both beams are considered small, with a large margin.

A more detailed analysis should be performed, comparing the results when assuming small deformation using a linear analysis and when assuming large deformations using a non-linear analysis. A linear analysis overestimates the deformations compared to a non-linear analysis but if the deformations are considered small, this overestimation is negligible.

### 4.7.2 Parametric study

A parametric study has been performed by comparing the response of a structure that is stiff enough to be considered rigid to an elastic structures with different lengths and stiffness'. To be able to include all frequencies in the comparison, one single value for each configuration is necessary. This value has been chosen to be the variance of the response spectra in waves. That is the integral of the response spectra of the structure in waves, i.e. the RAO squared times the wave spectra. A large variance indicates that high peak responses occur, possibly at the corresponding eigenfrequencies.

The lengths have been varied from 200 to 550 meter, the stiffness' have been varied between $(4-192) \cdot 10^{11} \text{Nm}^2$ which corresponded to a first wet elastic eigenfrequency of between 0.19 and 15.5 rad/s. This resulted in 135 data points which are plotted in figure 4.9 and normalized to the peak frequency $\omega_p$. The Bretschneider spectra with a peak period of 12 s corresponds to a peak frequency of 0.52 rad/s. The quasi-static response corresponds to the response of the shortest and stiffest beam i.e. the beam with the highest eigenfrequency.
Chapter 4. Simplified hydroelastic analysis

Figure 4.9: The quasi-static response normalized with the hydroelastic responses plotted against the first wet elastic eigenfrequency normalized with the peak frequency of a Bretschneider spectra of $H_s = 8m$ and $T_p = 12s$.

It is seen that when the first wet elastic eigenfrequency is about five times the peak frequency, or above, the response may be assumed quasi-static. If the first wet elastic eigenfrequency is lower than five times the peak frequency the response is assumed to be hydroelastic. Elastic responses occur in the hydroelastic cases around their respective first wet elastic eigenfrequency. Also at these elastic responses high stresses are observed, since deformations and stresses are related as described by equation (4.11).

Figure 4.10: Bretschneider spectra of $H_s = 8m$ and $T_p = 12s$, with the $\omega_p$ marked.

A wave spectra consist of many wave frequencies as seen in figure 4.10 and since it does so it could be discussed which frequency to compare with. Here the peak frequency has been chosen since it is very easy to observe in such a spectra.
4.7.3 Bending and shear stresses

Bending moments and shear forces are easily determined using Euler beam theory. The bending moment, when using Euler beam elements, is proportional to the second derivative of the deflection, and the shear forces are proportional to the first derivative of the bending moment, as

\[
M = EI \cdot \frac{d^2 z}{dx^2}
\]

(4.11)

\[
Q = -\frac{dM}{dx} = -EI \cdot \frac{d^3 z}{dx^3}
\]

(4.12)

where \( M \) is the bending moment, \( Q \) is the shear force, \( E \) is Young’s modulus, \( I \) is the second moment of area, \( z \) is the vertical deflection and \( x \) corresponds to the length direction of the beam. The maximum bending stress is given by

\[
\sigma = \frac{M}{I} (z + z_{NA})
\]

(4.13)

where \( z \) is the vertical coordinate and \( z_{NA} \) is the neutral axis of the beam. The stress is maximum at the top and bottom of the beam since it is symmetric with respect to the neutral axis. The \( z \)-dependence becomes \( z + z_{NA} = z_{beam}/2 \) where \( z_{beam} \) is the height of the beam. The maximum average shear stress is the maximum shear force, \( Q \), divided by the cross sectional area of the beam, \( A_{xs} \).

\[
Q_{max} = \text{max}(|Q|)
\]

(4.14)

\[
\tau_{max} = \frac{Q_{max}}{A_{xs}}
\]

(4.15)

The shear stress varies across the cross section and a more thorough analysis of the stress distribution should be performed to determine the maximum shear stress. The maximum bending stress and maximum averaged shear stress for the two-dimensional model is given in Table 4.2.

| Table 4.2: Maximum bending stress and maximum averaged shear stress of a beam. |
|---------------------------------|----------------|----------------|
| beam no.1 | beam no.2 | unit |
| Max. bending stress | 82 | 115 | MPa |
| Max. avg. shear stress | 6.7 | 4.2 | MPa |

These maximum values are observed in the vicinity of the first elastic natural frequency.
The maximum moment is observed in the middle of the beams and the maximum averaged shear stress is observed at about a quarter length from each beam ends. The yield strength of normal marine steel is at a minimum of 235 MPa according to (DNV-OS-B101, 2012). The maximum stress is at a reasonable level below the yield strength for both beams, for a wave amplitude of 1 meter.

Although, the stresses for beam no. 2 are observed at a frequency of about 2 rad/s which corresponds to wave lengths of about 15 meters. If the wave height is 5% of the wave length as previously assumed, the wave height is 0.75 meters which corresponds to a wave amplitude of 0.375 meters. For such a wave, the deformation amplitude would end up at roughly half a meter at the ends.

It should be noted that these result are only preliminary due to the sometimes harsh idealisations within the model. A more detailed model must be used to analysis the stresses and deformations at specific sea spectrum to conclude that the structure can withstand the specified weather conditions.

4.8 Conclusions and discussion

The parametric study concluded that the loading condition may be assumed quasi-static if the first wet elastic eigenfrequency is about five times the peak frequency in irregular waves.

The resonance amplitudes are largely determined by the structural and hydrodynamic damping properties. Both of these properties are very difficult to determine and they depend on a number of parameters and must be determined in experiments. A small change in damping will for lightly damped systems give rise to a large change in amplitudes around the natural frequencies. If the first elastic natural frequency inevitably ends up within the sea spectra of interest it is very important to have good knowledge these parameters so that the resonance amplitudes are controlled.

Since the first wet elastic natural frequency is within five times the peak frequency of the sea spectra for both beams a hydroelastic analysis must be performed in order to account for the hydroelastic effects of the structure. For beam no. 2 the first wet elastic natural frequency is almost within the sea spectra of interest and resonance may be
observed around this frequency. For beam no. 1 the first wet elastic natural frequency is outside the sea spectra of interest, thus lower deformations and stresses are observed compared to beam no. 2. Since these result are preliminary they should only indicate that the concept may resist the wave loads, and a more detailed model must be created for the real structure according to design level 3 to obtain more reliable results.

If the wave forces on the beams was included would the damping increase and the resonance amplitude decrease, the amplitude of the first elastic mode would also be reduced due to a more distributed loading condition. The model with the given idealisations resemble the structure in a decent way at wave lengths in about the same length as the beam, but when several wave lengths equal the beam length, i.e. at higher frequencies, does the model probably over estimate the deformations. A more accurate modelling method would be to model the wave forces on the beams as well as on the nodes. Inclusion of the tower and nacelle might alter the natural frequency slightly and local eigenfrequencies is be expected to arise.

Suggestions of future work would be to model the structure in a commercial FE-solver so that the stresses, deformations and motions may be determined for the real structure instead of a two-dimensional beam. The damping properties has to be investigated thoroughly since this is very important to know around elastic natural frequencies. Either Morison’s equation or the more advanced potential theory coupled with a FE-solver should be used. The latter could correspond to DNV design level 3, i.e. a very detailed design. By changing the position of the beam’s neutral axis it should be investigated if it’s possible to e.g. minimize the deformation and stresses around the first elastic natural frequency.
Chapter 5

Concluding remarks

5.1 Overall conclusions

In this thesis the degree of hydroelasticity has been investigated for a large floating wind turbine. The overall conclusions drawn from this study is:

- If the first wet elastic natural frequency is higher than 5 times the peak frequency of a sea spectra or roughly 2-5 times the wave frequency in regular waves, the loading condition may be assumed quasi-static, else a hydroelastic assumption must be assumed. The actual limit must be further investigated.

- If the first wet elastic natural frequency is within the sea spectra, resonance will occur, which should be avoided.

- Global hydroelasticity is seldom accounted for in the design of floating wind turbines and DNV doesn’t mention it in their offshore standard (DNV-OS-J103, 2013). However it may not be relevant in most of the cases, but even so; knowledge of if hydroelasticity is relevant to the concept should be known.

- The most promising computer programs for analysing global hydroelasticity with is ANSYS if Morison’s equation may be used or Wamit coupled to a FE-model if diffraction must be taken into account.

- The semi-flexible model used by GVA in the report analysing one of Hexicon’s platform was found to be insufficient and didn’t capture the hydroelastic behaviour correctly.

- These preliminary resulting stresses is within a reasonable level below the minimum recommended yield strength of steel. However the idealisations of the model are sometimes very harsh and the result may not be directly transferred to the real structure.
5.2 Contributions

What I have contributed with is to summarize different techniques of determining the degree of hydroelasticity for a semi-submersible and also conducted a parametric study of this. Also a compilation of how scientists nowadays perform hydroelastic calculations, and pros and cons for their respective techniques. A compilation of several floating wind turbine that are installed or in the concept phase has been conducted, with the main focus on the modelling techniques of the wave loadings and structural motions for these.

I have conducted a review of possible computer programs that may be used when a hydroelastic analysis needs to be performed.

Recommendations of different level of detail when designing a floating wind turbine has been presented which are taken directly from the DNV design levels. Within these have hydroelasticity been included, which was lacking from the original recommendations.

I have developed a simplified hydroelastic model that is able to simulate a semi-submersible which is built up by two nodes interconnected by a beam in a hydroelastic manner. This model has been used in the parametric study as well as for determining deformations and stresses for the structure.

I have reviewed the report by GVA which analysed one of Hexicon’s platforms.

5.3 Discussion of the assumptions

- Linear waves are assumed; so that Airy wave theory may be used. Airy theory is the simplest wave theory and also the most used because of its simplicity and its ability to produce irregular waves with.
  
  - This assumption is widely used and accepted in the industry.

- Small bending deformations are assumed; so that linear beam theory may be used.
  
  - A comparison in chapter 4 concluded that there are several rule-of-thumbs of determining whether there are small or large deformations occurring. All of these rule-of-thumbs indicated that small deformation was a reasonable assumption and linear beam theory may be used.
• The beams are modelled as one-dimensional Euler beams with shear deformation and rotary inertia neglected, which is a reasonable assumption for thin beams.

  – As a rule-of-thumb, thin beams are beams which have a height over length ratio of 1/10 or lower. Both the analysed beams are denoted thin beams by this definition.

• Deep water is assumed; To simplify the velocity potential equation (and because the platform is supposed to be situated in deep water for most of waves).

  – An investigation of the deep water assumption has been performed in section 4.5.3 which concluded that the assumption is adequate for wave frequencies of 0.5 rad/s or higher. Lower frequencies than this leads to an underestimation of the wave loads. The largest waves observed according to a Bretschneider spectra with $H_s = 8$ meters and $T_p = 12$ seconds have a wave frequency of about 0.4 rad/s, thus the load from these waves are underestimated slightly.

• The wave forces are assumed to correspond to Morison’s equation.

  – The comparison between Morison’s equation and diffraction theory in appendix A.1 concluded that Morison’s equation is valid if the diameter of the node over the wave length is less than 1/7. For a 10 meter node this means wave lengths larger than 70 meter, i.e. wave frequencies of 0.94 rad/s or less. In other words; Morison’s equation is valid for large waves where the largest forces occur and not at smaller waves.

• The added mass has been assumed to be equal the displaced volume times the density of the water, which is equal to the added mass at large frequencies.

  – This has been done to simplify the the analysis. A frequency dependant added mass should be included in a more detailed model. This would slightly change the deformations, stresses as well as the natural frequencies of the structure.

• The wave forces are only modelled on the nodes and neglected on the beams.

  – Wave loads modelled on the beams would decrease the amplitude of the elastic modes. With this included, the deformation would change more than just slightly. For future work, this would be the first thing to change.
• The nodes including towers and nacelles are modelled as point masses on the first
and last beam element.

  – If the model included the towers and nacelles would local eigenfrequencies be
  observed corresponding to their mass distribution, stiffness and length and
  local stresses would be observed in the interconnection between the tower and
  node. The bending stiffness of the nodes here equals the bending stiffness for
  the beam, which is wrong. However since the nodes are only a small part of
  the beam this assumption is adequate.

• A free-free boundary condition is assumed, which is a common boundary condition
  for structures in waves and according to (Bishop and Price, 1979).

  – This assumption may be questionable. Changing the boundary condition so
  that the beams are clamped to the nodes and that the nodes are free to move
  might alter the observed stresses, especially in the boundary between the
  node and the beam. This should be looked into in a future analysis.
Appendix A

Theoretical background

A.1 Morison’s equation vs. Diffraction

Two very common methods of calculating the hydrodynamic loads are Morison’s equation and diffraction theory. Comparison between the use of these theories have been examined by e.g. Chakrabarti (1987) and Kvittem et al. (2012). They have both created figures to assist the choice of theory in the design of offshore structures, shown in figure A.1. In the figure the different wave heights, $H$, over diameter of the nodes, $D$, has been plotted as a function of the wave height over wave length, $\lambda$, according to the legend. The wave height is assumed to be 5% of the wave length for all waves. This is well below the deep water wave breaking limit where the wave height is about 14% of
the wave length. The black vertically dashed line and the green horizontally dashed line are the same in both figures, thus, the wave breaking limit/deep water breaking wave curve is the same in both figures.

Figure A.1 shows that for wave heights over diameter of 0.36 – 1.2, corresponding to diameters over wave lengths smaller than 1/7, the nodes may be analysed using Morison’s equation. For wave heights over diameter of 0.15 – 0.35, corresponding to diameters over wave lengths larger than 1/7, the nodes must be analysed using diffraction theory. Hence, Morison’s equation is according to valid if the diameter over wave length is smaller than 1/7.

The figures are only valid for bottom mounted cylinders but it may give a good approximation to floating cylinders interconnected with beams (Kvittem et al., 2012), as this structure is.

If the node is 10 meter in diameter, then waves longer than 70 meter corresponds to a loading frequency of 0.94 rad/s or less. In storm conditions Morison’s equation may then be used in almost the entire frequency range, since the wave frequencies for the dominating part of the waves varies in between 0.4 – 1.0 rad/s. The main contributions to motions and stresses would come from the largest waves, which in this spectra are at about 0.5 – 0.6 rad/s and in this region Morison’s equation can accurately describe the wave forces. The structure is designed for these largest waves and will thus withstand the smaller waves, those waves where Morison’s equation doesn’t describe the forces accurately enough.
A.2 Wave theory

Waves created by the wind are irregular by nature and therefore it is not sufficient to only study regular waves. According to linear signal theory, the sum of linear waves are also linear in nature, which means that it is possible to create irregular waves by superimposing regular waves with different amplitude, frequency and phase shown in figure A.2. This is unlike Stoke’s waves where the superimposed waves all are of the same phase. This makes it very easy to create irregular waves based on linear wave theory. This is not the case with any of other wave theory. Irregular waves are defined by a wave spectra describing the density of the various regular wave components included in the irregular waves, as shown in figure A.3. If the wave spectra is defined as $S_\xi(\omega_\xi)$

![Figure A.2: Superposition of regular waves to create irregular waves, taken from (Faltinsen, 1990).](image)

![Figure A.3: Bretschneider wave spectra, taken from (Journee and Pinkster, 2002).](image)
then the wave amplitude is determined by

$$\zeta_{a,n} = \sqrt{2 \cdot S_\zeta(\omega_n) \cdot d\omega.} \quad (A.1)$$

when $\Delta \omega \to d\omega$. The peak frequency $\omega_p$ is where the spectra has its largest value and is a parameter often used to define a wave spectrum, such as the Bretschneider spectrum which definition is shown below.

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left( -\beta \left( \frac{\omega_p}{\omega} \right) \right) \quad (A.2)$$

where $\alpha$ and $\beta$ are constants, and $g$ is the gravitational coefficient. ISSC$^{19}$ have modified the Bretschneider spectra and defined it using the significant wave height, $H_s$, and zero-crossing period, $T_z$, according to

$$S_{ISSC}(\omega) = \frac{124 \cdot H_s^2}{T_z^4 \omega^5} \exp \left( -\frac{494}{\omega \cdot T_z^2} \right) \quad (A.3)$$

The Bretschneider spectrum is an empirical formula that describes the distribution of energy for each frequency of a fully developed sea state. The definition of a fully developed sea state is that the wind has blown over a large area and for a long period of time. Sea states that are not fully developed must be modelled using other wave spectra’s e.g. JONSWAP$^{20}$ spectrum (Stewart, 2013).

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$^{19}$International Ship and Offshore Structures Congress
$^{20}$Joint North Sea Wave Project
A.3 Derivation of forces

A.3.1 Wave forces

The wave coefficients are determined according to Morison’s equation, eq. (4.5) in section 4.5.2, but with a linear drag-term according to (4.7). The wave forces are here presented as a dynamic system as:

\[
F^{\text{wave}} = A^w \ddot{u} + B^w u + C^w \zeta
\]

where the coefficients \( A^w, B^w \) and \( C^w \) is the added mass, the damping, and stiffness term and \( \ddot{u}, u, \zeta \) is the wave particle acceleration, wave particle velocity and effective wave amplitude, respectively. The wave force components in the hydrodynamical model corresponding to the coefficients \( A^w, B^w \) and \( C^w \) are described in detail in section 4.5.2 by equations (4.3), (4.6), (4.7) and (4.8) and are given in respectively \( x \)- and \( z \)-direction as:

\[
\begin{align*}
A^w_x &= C_a \rho V \\
A^w_z &= C_a \rho \frac{2\pi}{3} \left( \frac{D}{2} \right)^3 \\
B^w_x &= \frac{1}{2} C_d \rho D T \sqrt{\frac{8}{\pi}} \sigma_u \\
B^w_z &= \frac{1}{2} C_d \rho \frac{D^2}{4} \sqrt{\frac{8}{\pi}} \sigma_u \\
C^w_x &= 0 \\
C^w_z &= \rho A_w
\end{align*}
\]

\( C_a \) is the added mass in respective directions, \( V \) is the volume of the submerged body, \( C_d \) is the drag coefficient in respective direction, \( D \) is the diameter of the diameter of the node, \( T \) is the draught of the node, \( \sigma_u \) is the standard deviation of the water particle velocity and \( A_w \) is the area about the stillwater level.

The effective wave amplitude is the effect of the wave amplitude has on a structure at a certain depth, and it is only applied in the vertical direction. The horizontal forces are denoted \( F^{\text{wave}}_x \) and the vertical forces are denoted \( F^{\text{wave}}_z \). These forces are only applied at the nodes since the forces on the beams has been neglected in this analysis. The particle acceleration, particle velocity and effective wave amplitude are derived from the velocity potential, which is described below. The velocity potential is

\[
\phi = \frac{i \zeta a g \cosh(k(z+h))}{\omega \cosh(kh)} e^{kx} e^{i\omega t}
\]
where \( z \) is the vertical distance to the force’s point of attack, \( h \) is the depth of the sea, \( k \) is the wave number, \( \omega \) is the wave frequency, \( \zeta_a \) is the wave amplitude, \( g \) is the gravitational constant, \( i \) is the imaginary number which describes the phase, \( x \) is the spatial variable and \( t \) is the time variable. Assuming deep water (the depth over wavelength is larger than one half) this expression may be simplified as follows.

\[
\frac{h}{\lambda} > 0.5 \rightarrow kh = 2\pi h/\lambda > \pi \tag{A.9}
\]

The cosh-terms in equation (A.8) reduces it to

\[
\frac{\cosh(k(z+h))}{\cosh(kh)} = \frac{\cosh(kz)\cosh(kh) + \sin(kz)\sinh(kh)}{\cosh(kh)} = \cosh(kz) + \sinh(kz)\tanh(kh) = \begin{cases} 
\tanh(kh) \approx 1 & \text{for } kh > \pi \\
\cosh(kh) + \sinh(kh) = e^{kz} 
\end{cases} \tag{A.10}
\]

The velocity potential valid for deep water is then

\[
\phi = \frac{i \zeta_a g}{\omega} e^{kx} e^{kx} e^{i\omega t} \tag{A.11}
\]

A detailed description of the velocity potential in water waves and the derivation of the boundary conditions is given in Garme (2011). By using the dynamic free surface boundary condition

\[
\frac{\partial \phi}{\partial t} + g \zeta = 0 \tag{A.12}
\]

the effective wave amplitude may be derived.

\[
\zeta = -\frac{1}{g} \frac{\partial \phi}{\partial t} = \frac{1}{g} i \frac{\zeta_a \omega g}{\omega} e^{k} e^{kx} e^{i\omega t} = \zeta_a e^{kx} e^{i\omega t} \tag{A.13}
\]

The wave particle velocity is described by the gradient of the velocity potential

\[
u = \nabla \phi \tag{A.14}
\]

and the wave particle acceleration by

\[
\dot{u} = \nabla^2 \phi. \tag{A.15}
\]
The wave particle velocity in deep water is thus
\[ u = \frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial z} = i\zeta agk \omega e^{kz} e^{kx} e^{i\omega t} = \left\{ gk = \omega^2 \right\} = i\omega \zeta a e^{kz} e^{kx} e^{i\omega t} \]  
(A.16)
and the wave particle acceleration is
\[ \dot{u} = \frac{\partial u}{\partial t} = -\omega^2 \zeta a e^{kz} e^{kx} e^{i\omega t}. \]  
(A.17)

The wave forces on one of the nodes are shown in figure A.4.

**The vertical wave force** which is applied at the bottom of the nodes at a draught \(-T\) may be expressed using equation (A.4), (A.13), (A.16) and (A.17) as follows
\[ F_{\text{wave}}^z = (-\omega^2 A^w_z + i\omega B^w_z + C^w_z)\zeta a e^{-kT} (e^{kx_1} + e^{kx_2}) e^{i\omega t} \]  
(A.18)
where \(A^w_z, B^w_z\) and \(C^w_z\) is the added mass, damping and stiffness term in the vertical direction, respectively, and \(e^{kx_1} + e^{kx_2}\) comes from the force being applied at the two nodes, at \(x_1 = -L/2\) and at \(x_2 = L/2\). The term \(e^{i\omega t} (e^{kx_1} + e^{kx_2})\) can be rewritten using Euler’s formula in complex analysis, where the real part of this term is
\[ \cos(kx_1 + \omega t) + \cos(kx_2 + \omega t). \]  
(A.19)
This in turn can be rewritten using the sum-to-product rule (Wikipedia, 2013a) as
\[ 2\cos(\omega t)\cos\left(\frac{k}{2}(x_2 - x_1)\right). \]  
(A.20)
By inserting $x_1 = -L/2$ and $x_2 = L/2$, the resulting term becomes
\[ 2\cos(\omega t)\cos\left(\frac{kL}{2}\right). \quad (A.21) \]

Rewriting $\cos(\omega t)$ to $e^{i\omega t}$ using Euler’s formula in complex analysis again, results in
\[ e^{kx_1} + e^{kx_2} = e^{-kL/2} + e^{kL/2} = 2\cos(\omega t)\cos\left(\frac{kL}{2}\right). \quad (A.22) \]

The concluding vertical wave force on the bottom of the nodes is
\[ F_{\text{wave}}^z = (-\omega^2 A^w_z + i\omega B^w_z + C^w_z)2\zeta_a e^{-kT}\cos\left(\frac{kL}{2}\right) e^{i\omega t}. \quad (A.23) \]

The vertical force will give rise to a bending deformation of the beam.

The **horizontal wave force** is described by
\[ F_{\text{wave}}^x = \int_{-T}^{0} dF_{x}^\text{wave}(z) \, dz \quad (A.24) \]

where $dF_{x}^\text{wave}$ is the force per unit length and is derived the same way as the vertical wave force, the only difference is that the $z$-variable goes from 0 to $-T$.

\[ F_{x}^\text{wave} = (-\omega^2 A^w_x + i\omega B^w_x)\zeta_a e^{kz}\cos\left(\frac{kL}{2}\right) e^{i\omega t} \quad (A.25) \]

where $A^w_x$ and $B^w_x$ is the added mass and damping term in the horizontal direction, respectively. The only $z$-dependance in equation (A.25) above is $e^{kz}$ and the integral in (A.24) therefore becomes
\[ \int_{-T}^{0} e^{kz} \, dz = \left[ \frac{e^{kz}}{k} \right]_{-T}^{0} = 1 - \frac{e^{-kT}}{k}. \quad (A.26) \]

The concluding horizontal wave force on the nodes is
\[ F_{x}^\text{wave} = (-\omega^2 A^w_x + i\omega B^w_x)\zeta_a \frac{1 - e^{-kT}}{k}\cos\left(\frac{kL}{2}\right) e^{i\omega t} \quad (A.27) \]

The horizontal force will not explicitly give rise to bending deformations of the beam, but it will give rise to a bending moment that will give rise to bending deformations.
The wave moment due to the horizontal force is

\[ M^\text{wave}_\theta = \int_{-T}^{0} dM^\text{wave}_\theta dz \]  \hspace{1cm} (A.28)

where

\[ dM^\text{wave}_\theta = (z - z_{NA}) \cdot dF^\text{wave}_x(z) \]  \hspace{1cm} (A.29)

and \( z_{NA} \) is the vertical coordinate of the neutral axis of the beam, shown in figure A.5, \((z - z_{NA})\) is the lever arm between the neutral axis and the horizontal wave force per unit depth \( dF^\text{wave}_x \), shown in figure A.4.

The only \( z \)-dependance in equation (A.29) is \((z - z_{NA})e^{kz}\), where \( z - z_{NA} \) is the lever arm of the wave moment and \( e^{kz} \) represent the wave force distribution. The integral in (A.28), excluding everything that is not \( z \)-dependant, becomes

\[
\int_{-T}^{0} (z - z_{NA})e^{kz} \, dz = \\
\{\text{Integrating by parts}\} \\
= \left[ (z - z_{NA}) \cdot \frac{e^{kz}}{k} \right]_{-T}^{0} - \int_{-T}^{0} \frac{1}{k} e^{kz} \, dz = \\
= \cdots = \frac{(k(T + z_{NA}) - 1)e^{-kT} - kz_{NA} - 1}{k^2} \\
\]

The wave moment may then be written as

\[
M^\text{wave}_\theta = 2\zeta a \left( \frac{(k(T + z_{NA}) - 1)e^{-kT} - kz_{NA} - 1}{k^2} \right) \left( -\omega^2 A^w_x + i\omega B^w_x \right) e^{i\omega t} \cos \left( \frac{kL}{2} \right) , \\
\]

(A.31)
The lever arm for the wave moment depends on the location of the neutral axis, and changing it may significantly alter the deformation and stress levels of the beam. The neutral axis should be below the stillwater level but the position must be investigated further if it’s possible to e.g. minimize the deformation and stresses around the first elastic natural frequency.

A.3.2 Structural forces

The structural forces are divided in rigid body forces and the forces on an elastic body. The forces on a rigid body are idealised as a dynamic system. The equation of motion is

\[ F_{n}^{\text{rigid}} = -A_n \ddot{z} - B_n \dot{z} - C_n z \]  \hspace{1cm} (A.32)

where \( A_n \) is the structural and added mass term, \( B_n \) is the hydromechanical damping term, \( C_n \) is the hydromechanical stiffness term in the global d.o.f. \( n = 0, 1 \) and \( \ddot{z}, \dot{z}, z \) is the acceleration, velocity and relative displacement of the structure, respectively. These hydromechanical coefficient are described in chapter 4.5.2 and appendix A.3.1. The global d.o.f, \( n \), for the rigid body are; \( n = 0 \) is the heave and \( n = 1 \) pitch. The forces on a elastic body are idealised as a dynamic system very similar to the rigid body motions. The equation of motion is thus

\[ F_{n}^{\text{elast}} = -A_n \ddot{z} - B_n \dot{z} - C_n z \]  \hspace{1cm} (A.33)

where \( A_n \) is the structural and added mass term, \( B_n \) is the structural damping term, \( C_n \) is the structural stiffness term in the global d.o.f \( n \geq 2 \). These structural force coefficients are determined using the orthogonality condition described in (Prof. Anderson, 2013) and (Preumont, 2013). The global d.o.f, \( n \), for the elastic body are equal or larger than two, where \( n = 2 \) is the first elastic mode, \( n = 3 \) is the second elastic mode etc.
A.4 Modelling method

The modal method consists in finding the mode shapes and the corresponding natural
frequencies \((\phi, \omega_n)\), calculate the response of each mode, also called modal response,
\((Z^m)\), for a given external force \((F^m)\) and superimpose each modal response to a total
response \((Z)\). This procedure is explained below. The equation of motion for the
structure can be described by

\[
A \ddot{Z} + B \dot{Z} + CZ = F^{\text{wave}}
\]  
(A.34)

where \(Z\) is the displacement vector, \(F^{\text{wave}}\) is the external force vector, \(A, B\) and \(C\)
are matrices containing the inertia, damping and stiffness attributes of the system for each
beam element division \(j\) i.e. local d.o.f. To find the modal response a transformation to
generalised or modal coordinates, \(Z^m\), \((m\,\text{for modal})\) is done

\[
Z = \phi Z^m
\]  
(A.35)

where \(\phi\) is a matrix containing the mode shapes of the structure. Using equation (A.35)
in (A.34) yields

\[
A\phi \ddot{Z}^m + B\phi \dot{Z}^m + C\phi Z^m = F^{\text{wave}}.
\]  
(A.36)

By multiplying on the left with \(\phi^T\) yields

\[
\phi^T A\phi \ddot{Z}^m + \phi^T B\phi \dot{Z}^m + \phi^T C\phi Z^m = \phi^T F^{\text{wave}}.
\]  
(A.37)

Substituting \(A^m, B^m\) and \(C^m\) which is the generalised inertia, damping and stiffness
matrices, respectively, and \(F^m\) which is the generalised force,

\[
\phi^T A\phi \ddot{Z}^m = A^m
\]  
(A.38)

\[
\phi^T B\phi \dot{Z}^m = B^m
\]  
(A.39)

\[
\phi^T C\phi Z^m = C^m
\]  
(A.40)

\[
\phi^T F^{\text{wave}} = F^m
\]  
(A.41)
Appendix A.5 Modelling method

in equation (A.37) yields

\[ A^m \ddot{Z}^m + B^m \dot{Z}^m + C^m Z^m = F^m. \]  

(A.42)

Equation (A.42) is called the generalised equation of motion. This is an energy balance rather than a force balance, according to the principles of virtual work. The generalised force is the work done by the external force on each mode. The mode shapes shown in figure A.6. For example the work done by an external force applied in the middle of the beam on the pitch mode (mode 1) is equal to zero since the deformation in the middle is zero, but the work done on the heave and first elastic mode are non-zero. Dependent on the loading condition some modes may be cancelled out while other modes are dominating the response.

**Figure A.6:** Mode shapes of mode 0 through 3. Mode 0 and 1 corresponds to rigid body modes (heave and pitch) and modes higher than or equal to 2 are elastic modes. All of the modes are normalized to unity.
The equation (A.42) forms a set of uncoupled equations that may be solved individually. By assuming a sinusoidal force, the response may be written

\[(C_m - \omega^2 A_m + i\omega B_m)Z_m = F_m\]  \hfill (A.43)

The transfer function for each mode \(n\) is

\[G_n(\omega) = \left[ C_n - \omega^2 A_n + i\omega B_n \right]^{-1}\]  \hfill (A.44)

and the modal response is the transfer function times the generalized force. The total response is thus a summation of the modal responses

\[Z_m(\omega) = \sum_{n=0}^{N} G_n(\omega)F_m.\]  \hfill (A.45)

To determine the displacements in Cartesian coordinates a retransformation from the generalised coordinate is made using equation (A.35) and (A.41) in (A.45).

\[Z(\omega) = \phi \sum_{n=0}^{N} G_n F_m(\omega) = \phi \phi^T \sum_{n=0}^{N} G_n(\omega)F^{wave}(\omega)\]  \hfill (A.46)

In theory all mode shapes are accounted for \((N \rightarrow \infty)\), but in practice a truncation at \(N\) number of modes are performed.
A.5 Stochastic linearisation

In Morison’s equation the drag term is quadratic with respect to the water particle velocity and in order to include the drag in a frequency domain analysis, linearization of it must be performed. The drag term from Morison’s eq, (A.47), is

\[ C_D \frac{1}{2} \rho D u |u| \]  

(A.47)

where \( u \) is the water particle velocity, \( \rho \) is the water density, \( D \) is the diameter of the structure and \( C_D \) is the drag coefficient. Linearization of the quadratic drag term may be done using stochastic linearization. Stochastic linearization consists of assuming the quadratic expression to be roughly equal to a constant, \( K(\sigma) \), proportional to the standard deviation of the water particle velocity of a given irregular sea response spectra \( \sigma_u \) times the water particle velocity, as

\[ u |u| \approx K(\sigma_u) \cdot u. \]  

(A.48)

According to Hartnett and Mullarkey (1999) the constant may be approximated to

\[ K(\sigma_u) = \sqrt{\frac{8}{\pi}} \sigma_u \]  

(A.49)

and the quadratic term is linearised as

\[ u |u| \approx \sqrt{\frac{8}{\pi}} \sigma_u u. \]  

(A.50)

An iteration process is needed to determine the standard deviation because the response spectra is not know beforehand. This type of linearization is widespread within the industry and is used in several simulation programs (ANSYS, 2013), (DNV-Wadam, 2010).
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